MATHEMATICS **Class: 7th**

Topic: Basics of Geometry (Line & Angles)

Line & Angles

Some Geometrical Terms

POINT: A point is an exact location; a finite dot represents a point. It is denoted by a capital letter A, B, P or S etc.

eg. P•

LINE SEGMENT: The straight line between two points A and B is called the line segment. It is denoted by $\overline{AB}$.

The points A and B are called the end points of the line segment $\overline{AB}$. A line segment has a definite length.

The distance between two points A and B is called length of the line segment $\overline{AB}$.

Or

A part of a line with two end points is called a line segment.

RAY: A line segment AB when extended indefinitely in one direction is the ray $\overrightarrow{AB}$

Ray $\overrightarrow{AB}$ has one end point A. Ray has no definite length.

Or

A part of a line with one end point is called a ray.

LINE: A line segment $\overline{AB}$ when extended indefinitely in both the directions is called the line. It is denoted by $\overleftrightarrow{AB}$.

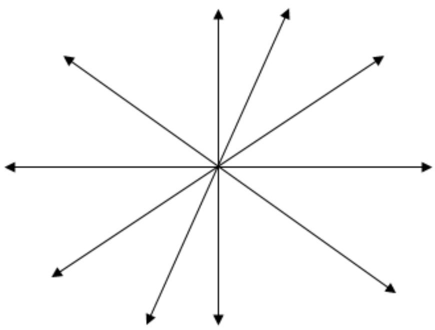
<u>*Note*</u>: 1) A line has no end points. 2) A line has no definite length

Incidence Axioms on Lines

(*i*) A line contains indefinitely many points.

(*ii*) Through a given point, indefinitely many lines can be drawn.

(*iii*) One and only one line can be drawn passing through two given points A and B.

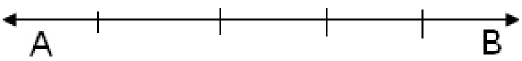


COLLINEAR POINTS

Three or more than three points are said to be collinear if they lie on the same straight line.

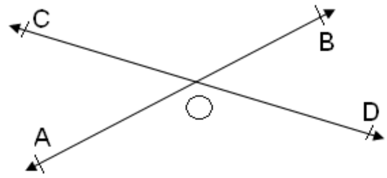
Or

Three or more than three points are said to be collinear if there is a line which contains all of them.



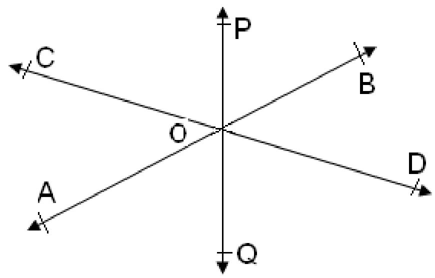
INTERSECTING LINES

Two lines are said to be intersecting if they meet or cut each other at only one point.



The common point is called their point of intersection. In figure the lines AB and CD intersect at a point O.

CONCURRENT LINES



Three or more lines intersecting at the same point are said to be concurrent.
In figure shown lines AB, CD, PQ pass through the same point O. Therefore they are concurrent.

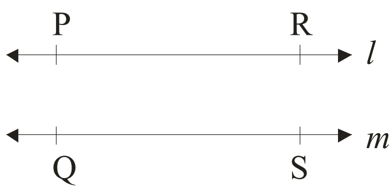
PARALLEL LINES

(*i*) Two or more lines are said to be parallel to each other if they do not meet / intersect each other.

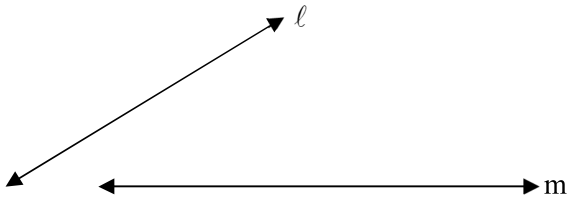
(*ii*) Two lines l and m in a plane are said to be parallel, if they have no point in common and we write $l \parallel m$.

Note: The distance between two parallel lines always remains the same throughout.

i.e. PQ = RS



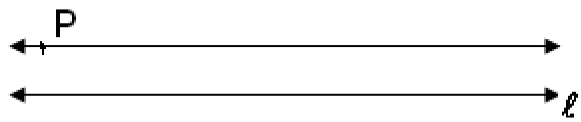
Some Results on Parallel Lines



(i) Two distinct lines cannot have more than one point in common.

Lines l and m will have only one point in common.

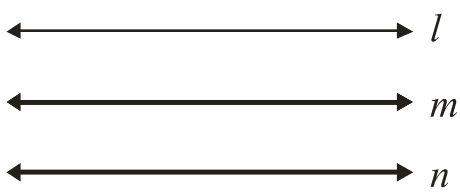
(ii) Two intersecting lines cannot be parallel to the same line.



If P is a point outside a given line l, then one and only one line can be drawn to pass through P and parallel to l.

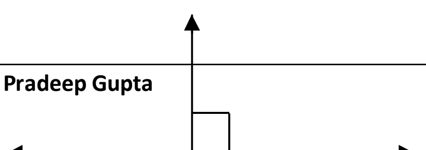
(iii) Two lines which are both parallel to the same line, are parallel to each other.

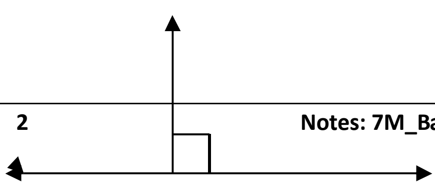
If $l \parallel n$ and $m \parallel n$. Then $l \parallel m$



Perpendicular Lines

Two lines are said to be perpendicular to each other if they make right angle, ie: they make an angle of 90° with each other

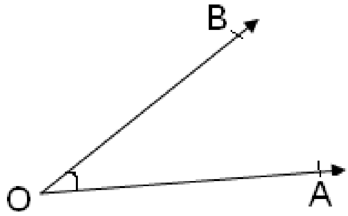




90° 90°

ANGLES AND THEIR PROPERTIES:

(1) **ANGLE:**

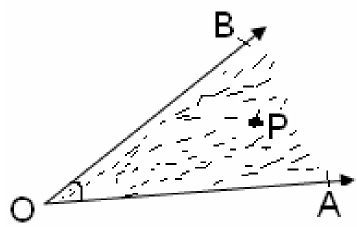


(*i*) When the two rays originate from the same point, an angle is formed.

(*ii*) Two rays OA and OB having a common originating point O, form an angle AOB, written as $\angle AOB$.

OA and OB are called the arms of the angle and O is called its vertex.

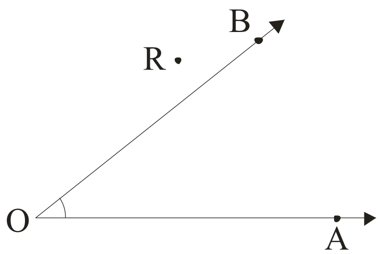
(2) INTERIOR OF AN ANGLE:



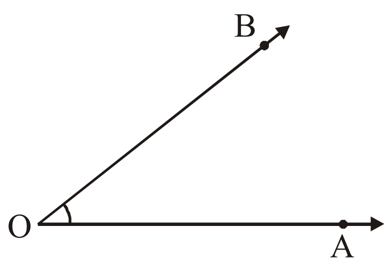
The interior of $\angle AOB$ is the set of all points in its plane, which lie on the same side of OA as B and also on the same side of OB as A, e.g. P is a point in the interior of $\angle AOB$.

(3) EXTERIOR OF AN ANGLE:

The exterior of an angle $\angle AOB$ is the set of all those points in its plane, which do not lie on the angle or in its interior. e.g. R is a point in the exterior of $\angle$ AOB.



Measure of an Angle



The amount of rotation of OB from OA is called the measure of $\angle AOB$, written as m$\angle AOB$.

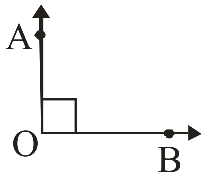
The unit of measure of an angle is ***degree.***

Note: It can be measured in Radians & Grades also.

Types of Angles:

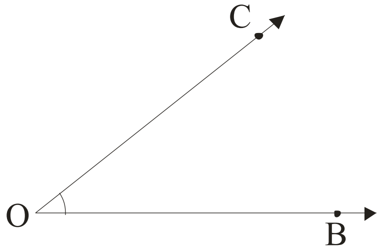
(1) Right Angle

An angle whose measure is 90º is called a right angle. $\angle AOB$ is a right angle.



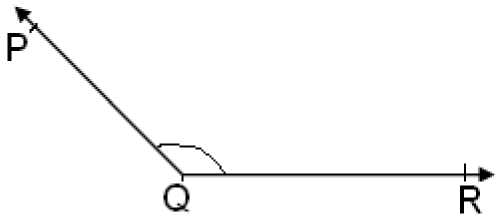
(2) Acute Angle

An angle whose measure is less than 90° but more than 0° is called an acute angle. $\angle COB$ is acute angle.



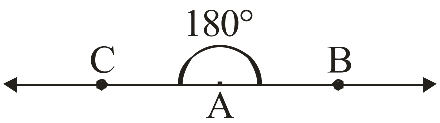
(3) Obtuse Angle

An angle whose measure is more than 90° but less than 180° is called an obtuse angle. $\angle PQR$ is an obtuse angle.



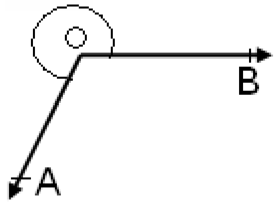
(4) STRAIGHT ANGLE

An angle whose measure is 180° is called a straight angle.



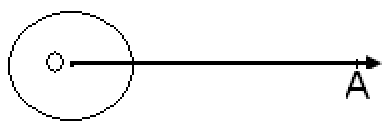
(5) REFLEX ANGLE

An angle whose measure is more than 180° but less than 360° is called a reflex angle.



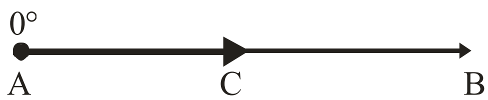
(6) COMPLETE ANGLE:

An angle whose measure is 360° is called a complete angle.



(7) ZERO ANGLE

An angle whose measure is 0° is known as zero angle.



Angle between ray AC and ray AB is zero degree.

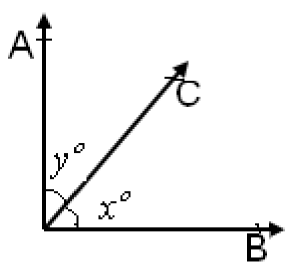
(8) COMPLEMENTARY ANGLES:

Two angles, the sum of whose measures is 90°, are complementary angles.

e.g. Angles of measure 50° and 40° are complementary angles,

because 50° + 40° = 90°

So we say that 50° and 40° are complement to each other. $\angle AOC$ & $\angle BOC$ are complementary angles if $x° + y° = 90°$

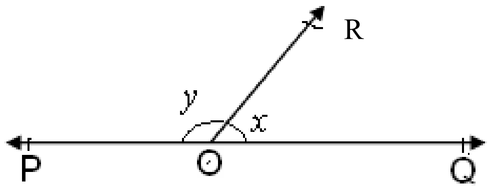


(9) SUPLEMENTARY ANGLES:

Two angles, the sum of whose measure is 180° are called the supplementary angles.

e.g. Angles of measure 115° and 65° are a pair of supplementary angles.

$\angle POR$ and $\angle QOR$ are supplementary if $x^\circ + y^\circ = 180^\circ$



Illustrative Examples

Example 1: Find the measure of an angle which is complement of itself.

Solution: Let the measure of the angle be x°.

The measure of its complement is given to be x°.

Since the sum of the measures of an angle and its complement is 90°

$\therefore \quad x^\circ + x^\circ = 90^\circ \Rightarrow 2x^\circ = 90^\circ \Rightarrow x^\circ = 45^\circ$

Example 2: In supplementary angles one is twice the other. Find the angles.

Solution: Two angles are called supplementary, if their sum is equal to 180°.

Let first angle be x°

Then the second angle will be $2x^\circ$

Hence, $x + 2x = 180^\circ$

$3x = 180^\circ$

$x = 60^\circ$

Hence, the first angle = 60° and the second angle = 120°

Example 3: Find the measure of an angle which forms a pair of supplementary angles with itself.

Solution Let the measure of the angle be x°. Then, by the given condition

$\therefore \quad x^\circ + x^\circ = 180^\circ \Rightarrow 2x^\circ = 180^\circ \Rightarrow x^\circ = 90^\circ$

Example 4: An angle is 16° more than its complement. What is its measure?

Solution: Let one angle = x°

Then second angle = $(x + 16)^\circ$

Now $x + (x + 16) = 90^\circ$

$\Rightarrow \quad 2x + 16 = 90^\circ$

$\Rightarrow \quad 2x = 90^\circ - 16^\circ = 74^\circ$

$\Rightarrow \quad x = 37^\circ$

$\therefore$ First angle = 37°

Second angle = 37° + 16° = 53°

Example 5: If the difference between two supplementary angles is equal to one right angle, then find the angles.

Solution: Let the two angles be x° and $(x + 90)^\circ$

Hence $x + (x + 90) = 180^\circ$

$\Rightarrow \quad 2x + 90^\circ = 180^\circ$

$\Rightarrow \quad 2x = 90° \quad \Rightarrow \quad x = 45°$

Thus the first angle is 45° and the second angle = 45° + 90°=135°

Example 6: An angle is equal to one-third of its supplement. Find its measure.

Solution: Let the measure of the required angles be $x°$. Then, the measure of its supplement is $(180 - x°)$.

It is given that; Angle $= \frac{1}{3}$ (its supplement)

$$\Rightarrow \quad x° = \frac{1}{3}(180 - x)°$$
$$\Rightarrow \quad 3x° = 180° - x°$$
$$\Rightarrow \quad 4x° = 180°$$
$$\Rightarrow \quad x° = 45°$$

Thus, the measure of the given angle is 45°.

Example 7: The measure of an angle is four times the measure of its supplementary angle. Find the angles.

Solution: Let the two angles be x and 4x.

$x + 4x = 180°$

$\therefore \quad 5x = 180° \quad \Rightarrow \quad x = 36°$

$\therefore$ The first angle = 36° and the second angle = 144°

Example 8: An angle is equal to five times its complement. Determine its measure.

SOLUTION: Let the measure of the given angle be $x°$. Then, the measure of its complement is $(90 - x)°$.

It is given that:

$\therefore$ Angle = 5 × Its complement

$$\Rightarrow \quad x = 5(90 - x)°$$
$$\Rightarrow \quad x = 450° - 5x$$
$$\Rightarrow \quad 6x = 450° \quad \Rightarrow \quad x = 75°$$

Thus, the measure of the given angle is 75°.

Example 9: Two supplementary angles differ by 34°. Find the angles.

SOLUTION Let one angle be $x°$. Then, the other angles is $(x+34)^o$. It is given that $x°$ and $(x+34)^o$ are supplementary angles.

$$\therefore \quad x° + (x+34)° = 180°$$
$$\Rightarrow \quad 2x° + 34° = 180°$$
$$\Rightarrow \quad 2x° = 180° - 34° = 146°$$
$$\Rightarrow \quad x° = 73°$$

Thus, two angles are of measures 73° and 73° + 34°= 107°.

Example 10: The supplement of an angle is one-third of itself. Determine the angle and its supplement.

SOLUTION: Let the measure of the angle be $x°$. Then, measure of its supplementary angle is (180-x) °,

It is given that $(180 - x)° = \frac{1}{3}x°$

$\Rightarrow \quad 3(180 - x)° = x°$

$\Rightarrow \quad 540° - 3x° = x°$

$\Rightarrow \quad 4x° = 540° \quad \Rightarrow \quad x° = 135°$

Thus, the measure of the angle is $135°$ and the measure of its supplementary is 180° – 135° = 45°

Example 11: Two supplementary angles are in the ratio 2:3. Find the angles.

SOLUTION: Let the two angles be $2x$ and $3x$ in degrees. Then,

$\therefore \quad 2x + 3x = 180°$

$\Rightarrow \quad 5x = 180° \quad \Rightarrow \quad x = 36°$

Thus, the measures of two angles are $2x = (2 \times 36)° = 72°$ and $3x = (3 \times 36)° = 108°$.

Example 12: Find the measure of an angle which is 24 more than its complement.

SOLUTION : Let the measure of the required angle be $x°$. Then, the measure of its complement = (90 - x)°.

$\therefore \quad x° - (90 - x)° = 24° \Leftrightarrow 2x° = 114° \Leftrightarrow x° = 57°$.

Hence, the measure of the required angle is 57°.

Example 13: Find the measure of an angle which is 32° less than its supplement.

SOLUTION: Let the measure of the required angle be $x°$.

Then measure of its supplement = (180-x)°.

$\therefore \quad (180 - x) - x = 32 \Leftrightarrow 2x = 148 \Leftrightarrow x = 74$

Hence, the measure of the required angle is 74°.

Example 14: Find the measure of an angle, if six times its complement is 12 less than twice its supplement.

SOLUTION: Let the measure of the required angle be $x°$

The, measure of its complement = $(90 - x)^o$.

Measure of its supplement $= (180 - x)^o$.

$\therefore \quad 6(90 - x)° = 2(180 - x)° - 12 \Leftrightarrow 540° - 6x° = 360° - 2x° - 12$

$\Leftrightarrow 4x = 192° \Leftrightarrow x = 48°$.

Hence, the measure of the required angle is 48°.

PRACTICE QUESTIONS

1. Find the complement of each of the following angles.

(i) 58^o (ii) 60^o (iii) $\frac{2}{3}$ of a right angle

2. Find the supplement of each of the following angles.

   (i) 63^o (ii) 138^o (iii) $\frac{3}{5}$ of a right angle

3. Find the measure of an angle which is

   (i) Equal to its complement, (ii) equal to its supplement.

4. If the difference between two supplementary angles is 60°, then find the angles.
5. Find the measure of an angle which is 36^o more than its complement.
6. Find the measure of an angle which is 25^o less than its supplement.
7. The measure of an angle is three times the measure of its supplementary angle. Find the angles.
8. Find the angle which is a) four times its complement b) five times its supplement.
9. Find the angle whose supplement is four times its complement.
10. What will be the angle between the hands of a clock at the following time?

    (i) 6 A.M (ii) 9 P.M

11. Two supplementary angles are in the ratio 2:5. Find the angles.
12. Two complementary angles in the ratio 1:7. Find the angles.
13. P lies in the interior of $\angle BAC = 70°$ and the $\angle BAP = 42°$, determine the measure of the $\angle PAC$.
14. An angle is 26° more than its complement. What is its measure?
15. Find the measure of an angle, if seven times its complement is 10^o less than three times its supplement.
16. If an angle is 28^o less than its complement, find its measure.
17. If an angle is 30^o more than one half of its complement, find the measure of the angle.
18. Two supplementary angles are in the ratio 4:5. Find the angles.
19. Two supplementary angles differ by 38^o. Find the angles.
20. An angle is equal to 6 times its complement. Determine its measure.